

# Synchronicity From Synchronized Chaos

Gregory S. Duane\*

*Dept. of Atmospheric and Oceanic Sciences  
UCB 311*

*University of Colorado  
Boulder, CO 80309-0311*

*and*

*Thunder Bay Regional Research Institute  
290 Munro St.*

*Thunder Bay, ON Canada  
P7A 7T1*

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The dynamical systems paradigm of synchronized chaos may realize the philosophical notion of “synchronicity”. Effectively unpredictable chaotic systems commonly exhibit a predictable relationship when they are coupled through only a few of many variables, a relationship that can be highly intermittent in realistic configurations, as with philosophical “synchronicities”. The notion of synchronicity between matter and mind is realized dynamically if mind is analogized to a computer model assimilating data from observations, as in meteorology. Synchronicities are *meaningful*, as philosophically required, if meaningfulness is related to internal coherence. Internal synchronization indeed facilitates synchronization with an external system as illustrated for systems that exhibit oscillons, primitive time-varying coherent structures. Synchronicity-as-synchronized-chaos has useful implications both for neuroscience and for basic physics. In the former realm, it appears that representations based on synchronized chaotic oscillators are useful for global combinatorial optimization, as in perceptual grouping, and for fusion of alternative representations of an objective process, as may give rise to consciousness. Basic physical synchronicity is manifest in the non-local quantum connections implied by Bell’s theorem. The quantum world seems to reside on a generalized synchronization “manifold”, that one can either take as primitive or as evidence of a multiply-connected spacetime.

**Keywords:** synchronized chaos, on-off intermittency, machine perception, coherent structures, quantum nonlocality, microwormholes

## I. INTRODUCTION

Synchronization within networks of oscillators is widespread in nature, even where the mechanisms connecting the oscillators are not immediately apparent. One recalls the example of the synchronization of clocks suspended on a common rigid wall, a paradigm commonly attributed to Huygens. As with similar phenomena of fireflies blinking in unison, or female roommates synchronizing hormonal cycles, the patterns suggest a universally valid organizational principle that transcends any detailed causal explanation. Further from everyday experience, but perhaps related<sup>16</sup>, are the quantum mechanical harmonies between distant parts of a system that are not causally connected, involving also the observer’s choice of measurements as implied by Bell’s Theorem<sup>5</sup>.

Synchronous relationships that are difficult to explain causally have figured prominently in primitive cultures and certain philosophical traditions<sup>41</sup>. Typical of such ideas is the notion of “synchronicity” associated with Jung<sup>64</sup>, that has two essential characteristics beyond the simple simultaneous occurrence of corresponding events: First, the simultaneous occurrences or “synchronicities” must be isolated occurrences. Second, the synchronicities must be “meaningful”. The idea of synchronicity thus goes beyond the synchronization of oscillators in positing a new kind of order in the natural world, schematized by Jung and Pauli (Fig. 1) in their 1953 book *The interpretation of Nature and of the Psyche*<sup>30</sup>. A particularly important instance is the synchronization of matter and mind. In this view, mind is not slaved to the objective world, but tends to synchronize with it, based on limited exchange of information. Jung’s examples of synchronicity, and subjective perceptions of synchronicity generally, are commonly dismissed as the result of chance, but a minority opinion follows Pauli in asserting that a synchronistic order exists in the world alongside the causal one.

The study of coupled networks of oscillators in physical systems has focussed on regular oscillators with periodic limit-cycle attractors. Such models afford explanations for such surprising synchronous relationships as the one

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\* corresponding author: email address gregory.duane@colorado.edu

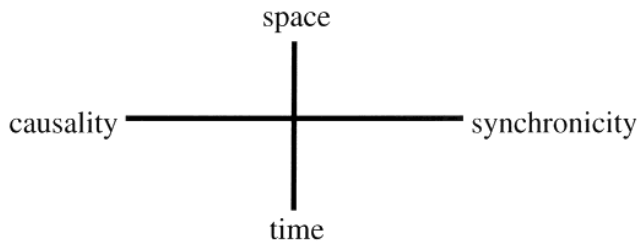


FIG. 1: Diagram constructed by Carl Jung, later modified by Wolfgang Pauli, to suggest relationships based on synchronicity as an “acausal connecting principle”, existing alongside causal relationships<sup>30</sup>.

observed by Huygens, but fall far short of Jung’s vision, as Strogatz observed in his popular exposition<sup>51</sup>.

While the synchronization of chaotic oscillators with strange attractors has become familiar in the last two decades, most work on such systems has examined engineered systems, primarily for application to secure communications, using the low-dimensional signal connecting the oscillators as a carrier that is difficult to distinguish from noise. However, a few examples of synchronized chaos in pairs of systems of partial differential equations that describe physical systems, coupled loosely, have also been given.

The point of this paper is to show that the synchronization of loosely coupled chaotic oscillators, as realized in nature, goes much further toward the philosophical notion of synchronicity than does synchronization of regular oscillators, if not actually reaching it. We begin by showing, in the next section, that the simple introduction of a time delay in the coupling between the systems can transform a situation of complete synchronization to one of isolated “synchronicities”. In Section III, we review previous work on an application of synchronized chaos to “data assimilation” of observations of a “real” system into a computational model that is intended to synchronize with truth. A parallel to the synchronization of matter and mind, as envisioned by Jung, allows us to introduce a working definition of “meaning”. In Section IV, a three-way relationship between two parts of a real system and a third system conceived as an observer is shown to satisfy the requirement for meaningfulness in synchronicities.

The objective rational basis for synchronicity that is put forward in this paper suggests applications of the new organizational principle to processes in the brain and in the physical world. In Section V, we discuss implications of synchronized chaos for neural systems, in view of contemporary ideas about synchronization as a binding mechanism in perception and consciousness. In Section VI, we argue that synchronized chaos can support quantum nonlocality and the Bell correlations in a realist interpretation, where the difference with classical physics is less fundamental, provided that one follows Einstein’s tradition in abandoning simple space-time geometry, but goes further in this direction than did he. The concluding section speculates on remaining gaps between our objective realization of the synchronicity principle and its original philosophical motivation.

## II. HIGHLY INTERMITTENT SYNCHRONIZATION IN LOOSELY COUPLED CHAOTIC SYSTEMS

Extensive interest in synchronized chaotic systems was spurred by the work of Pecora and Carroll<sup>43</sup>, who considered configurations such as the following combination of Lorenz systems:

$$\begin{aligned}
 \dot{X} &= \sigma(Y - X) & \dot{Y}_1 &= \rho X - Y_1 - X Z_1 \\
 \dot{Y} &= \rho X - Y - X Z & \dot{Z}_1 &= -\beta Z_1 + X Y_1 \\
 \dot{Z} &= -\beta Z + X Y
 \end{aligned} \tag{1}$$

which synchronizes rapidly: As  $t \rightarrow \infty$ ,  $Y_1(t) - Y(t) \rightarrow 0$ ,  $Z_1(t) - Z(t) \rightarrow 0$ . (Synchronization also occurs if the slave system is driven by the master  $Y$  variable instead of the  $X$  variable, but not if driven by the  $Z$  variable.)

Synchronization can also occur with weaker forms of coupling than the complete replacement of one variable by its corresponding variable as in (1), but degrades below a threshold coupling strength. Typically, synchronization degrades via on-off intermittency<sup>37</sup>, where bursts of desynchronization occur at irregular intervals, or as “generalized” synchronization<sup>47</sup>, where a strict correspondence remains between the two systems, but that correspondence is given by a less tractable function than the identity. As shown schematically in Fig. 2, as differences between the two systems increases, the correspondence changes from the identity to a smooth function that approximates the identity, to one given by a function that is nowhere differentiable. The last case is in fact common<sup>49</sup>.

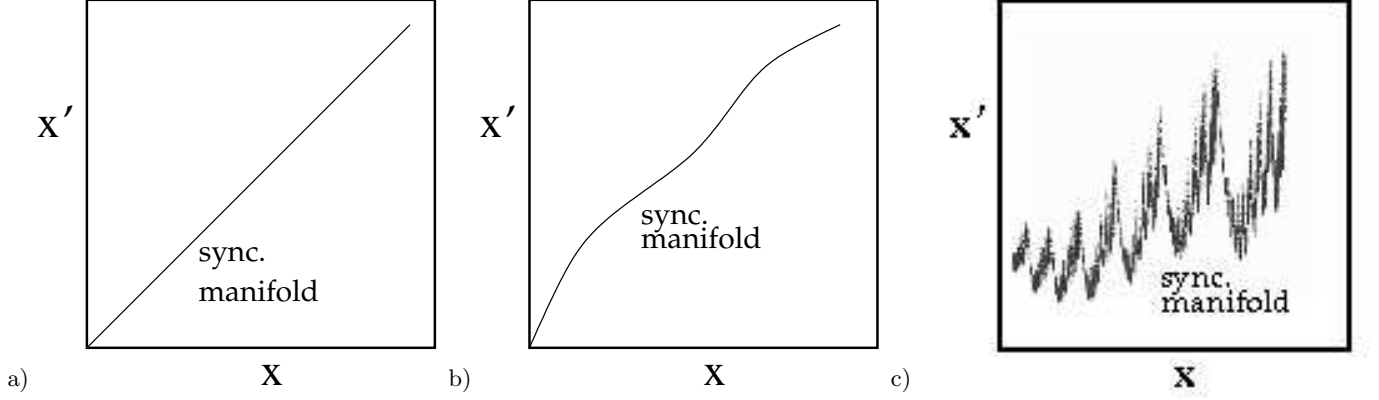


FIG. 2: Transition from identical to generalized synchronization, illustrated by the relationship between a pair of corresponding variables  $x$  and  $x'$ : Projection of the synchronization manifold onto the  $(x, x')$  plane are shown for (a) identical synchronization, (b) generalized synchronization with near-identical correspondence, (c) generalized synchronization with a correspondence function that is not smooth.

One can consider a large array of oscillators with synchronization among subsets or the entire array. In this context, the chaos synchronization phenomenon merges nicely with that of small world networks, or more generally, scale free networks in which the number of highly connected nodes decreases with the number of their connections according to a power law<sup>51</sup>. Randomly connected networks might be expected to synchronize more readily than regular networks that are connected only in local neighborhoods: the introduction of a few long-range connections can lead to a phase transition to long-range synchronization<sup>4,28,34,63</sup>.

While initial research on synchronized chaos was motivated by potential applications to secure communications, in applications to physical systems it is natural to consider forms of coupling that embody a time delay. If one extends chaos synchronization to the realm of naturally occurring systems, the delay in transmission ought to be described in terms of the same physics that governs the evolution of the systems separately. To a first approximation let us assume that the time scale of the delay is the same as some intrinsic dynamical time scale of each system. Consider the following configuration of two Lorenz systems, coupled through an auxiliary variable  $S$  that introduces a delay:

$$\begin{aligned}
 \dot{X} &= \sigma(Y - X) & \dot{X}_1 &= \sigma(Y_1 - X_1) \\
 \dot{Y} &= \rho(X - S) - Y - (X - S)Z & \dot{Y}_1 &= \rho(X_1 + S) - Y_1 - (X_1 + S)Z_1 \\
 \dot{Z} &= -\beta Z + (X - S)Y & \dot{Z}_1 &= -\beta Z_1 + (X_1 + S)Y_1 \\
 \dot{S} &= -\Gamma S + \Gamma(X - X_1)
 \end{aligned} \tag{2}$$

The system (2) is a generalization of the Pecora-Carroll coupling scheme (1) to a case with bidirectional coupling and where each subsystem is partially driven and partially autonomous.

As  $\Gamma \rightarrow \infty$  in (2), with  $\dot{S}$  finite,  $S \rightarrow X - X_1$ . In this limit, the system reduces to a bidirectionally coupled version of (1), which indeed synchronizes. In the general case of the coupled system (2) with finite  $\Gamma$ , the subsystems exchange information more slowly: if  $X$  and  $X_1$  are slowly varying, then  $S$  asymptotes to  $X - X_1$  over a time scale  $1/\Gamma$ . Thus  $\Gamma$  is an inverse time lag in the coupling dynamics.

Synchronization along trajectories of the system (2) is represented in Fig. 3 as the difference  $Z - Z_1$  vs. time, for decreasing values of  $\Gamma$ . For large  $\Gamma$ , the case represented in Fig. 3a, the subsystems synchronize. As  $\Gamma$  is decreased in Figs. 3b-d, corresponding to increased time lag, increasingly frequent bursts of desynchronization are observed, until in Fig. 3d (uncoupled systems) no portion of the trajectory is synchronized. The bursting behavior can be understood as an instance of on-off intermittency<sup>37,45</sup>, the phenomenon that may occur when an invariant manifold containing an attractor loses stability, so that the attractor is no longer an attractor for the entire phase space, but is still effective in portions of the phase space. Trajectories then spend finite periods very close to the invariant manifold, interspersed with bursts away from it.

The case of a coupling time lag that is of the same order as the prescribed physical time scale in the simple Lorenz system corresponds to  $\Gamma = 1$ , with behavior as in Fig. 3c. Although there is little trace of synchronization, the average instantaneous distance between the subsystems is less than it is in the uncoupled case. More interestingly,

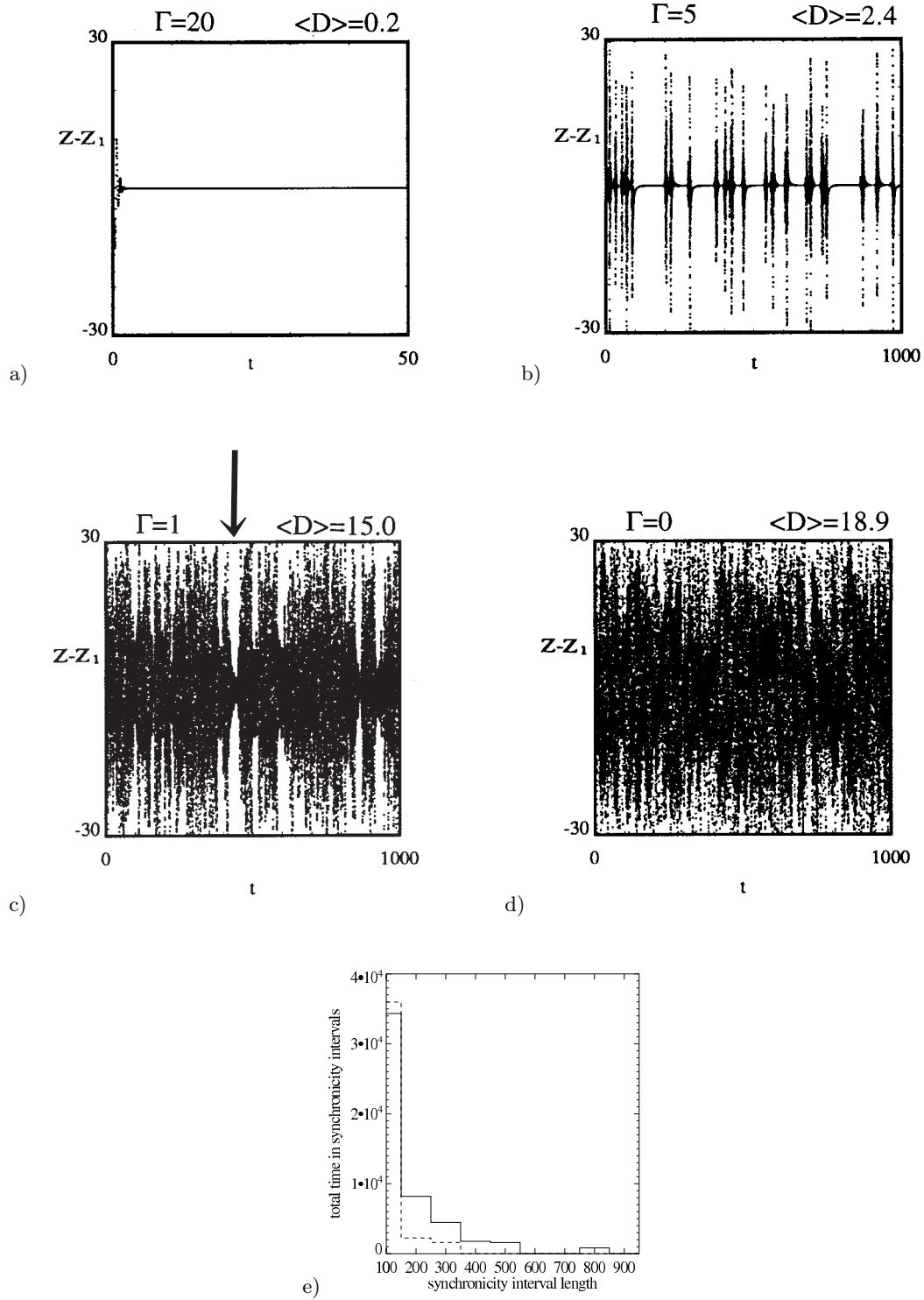


FIG. 3: The difference between the simultaneous states of two Lorenz systems with time-lagged coupling (2), with  $\sigma = 10.$ ,  $\rho = 28.$ , and  $\beta = 8/3$ , represented by  $Z(t) - Z_1(t)$  vs.  $t$  for various values of the inverse time lag  $\Gamma$  illustrating complete synchronization (a), intermittent or “on-off” synchronization (b), partial synchronization (c), and de-coupled systems (d). Average euclidean distance  $\langle D \rangle$  between the states of the two systems in  $X, Y, Z$ -space is also shown. A histogram of the lengths of periods of “synchronicity”, such as the one indicated by the arrow in (c), is shown in (e) for the time-delayed coupling case (solid line) and a case of two unrelated Lorenz trajectories (dashed line), where synchronicity intervals are periods during which  $|Z(t) - Z_1(t)| < 5$ .

there is a very short period of nearly complete synchronization. To study the commonality of such “synchronicities”, a histogram showing the number of such events of a given duration is plotted in Fig. 3e.

### III. MACHINE PERCEPTION AS CHAOS SYNCHRONIZATION

Computational models that predict weather include a feature not found in numerical solutions of other initial-value problems: As new data is provided by observational instruments, the models are continually re-initialized. This *data assimilation* procedure combines observations with the model’s prior prediction of the current state - since neither observations nor model forecasts are completely reliable - so as to form an optimal estimate of reality at each instant in time. While similar problems exist in other fields, ranging from financial modelling to factory automation to the real-time modelling of biological or ecological systems, data assimilation methods are far more developed in meteorology than in any other field.

The usual approach to data assimilation is to regard it as a tracking problem that can be solved using Kalman filtering or generalizations thereof. But clearly the goal of any data assimilation is to synchronize model with reality, i.e. to arrange for the former to converge to the latter over time. Thus the synchronously coupled systems of the previous section are re-interpreted as a “real” system and its model. In the system (1), for instance, we imagine that the world is a Lorenz system, that only the variable  $X$  is observed, and that the observed values are passed to a perfect model.

It is necessary to show that the synchronization phenomenon persists as the dynamical dimension of the model is increased to realistic values. Chaos synchronization in the sort of models given by systems of partial differential equations that are of interest in meteorology and other complex modelling situations has indeed been established. Pairs of 1D PDE systems of various types, coupled diffusively at discrete points in space and time, were shown to synchronize by Kocarev *et al.*<sup>32</sup>.

Synchronization in geofluid models that are relevant to weather prediction was demonstrated by Duane and Tribbia<sup>14,15</sup>. The models<sup>58</sup> are given in terms of the streamfunction  $\psi(x, y, i, t)$  in a 2-layer ( $i = 1, 2$ ) channel, contours of which are streamlines of atmospheric flow, as shown in Fig. 4, and a derived field variable, the potential vorticity  $q_i \equiv f_0 + \beta y + \nabla^2 \psi_i + R_i^{-2}(\psi_1 - \psi_2)(-1)^i$ , with constants as defined in the reference. The dynamical equation for each model is

$$Dq_i/Dt \equiv \partial q_i/\partial t + J(\psi_i, q_i) = F_i + D_i \quad (3)$$

where the Jacobian  $J(\psi, \cdot) \equiv \frac{\partial \psi}{\partial x} \frac{\partial \cdot}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \cdot}{\partial x} = v_y \frac{\partial \cdot}{\partial y} + v_x \frac{\partial \cdot}{\partial x}$  gives the advective contribution to the Lagrangian derivative  $D/Dt$ . The equation (3) states that potential vorticity is conserved on a moving parcel, except for forcing  $F_i \equiv \mu(q_i^* - q_i)$  and dissipation  $D_i$  as defined by Duane and Tribbia<sup>15</sup>. The forcing induces a relaxation to a jet-like background flow  $\psi^*$  (Fig. 4a,b) with  $q^* \equiv q(\psi^*)$ .

Two models of the form (3),  $Dq^A/Dt = F^A + D^A$  and  $Dq^B/Dt = F^B + D^B$  were coupled diffusively through a modified forcing term  $F_{\mathbf{k}}^B = \mu_{\mathbf{k}}^c[q_{\mathbf{k}}^A - q_{\mathbf{k}}^B] + \mu_{\mathbf{k}}^{ext}[q_{\mathbf{k}}^* - q_{\mathbf{k}}^B]$ , where the flow has been decomposed spectrally and the subscript  $\mathbf{k}$  on each quantity indicates the wave number  $\mathbf{k}$  spectral component. The two sets of coefficients  $\mu_{\mathbf{k}}^c$  and  $\mu_{\mathbf{k}}^{ext}$  were chosen to couple the two channels in some medium range of wavenumbers.

It was found that the two channels rapidly synchronize if only the medium scale modes are coupled (Fig. 4), starting from different initial flow patterns. For unidirectional coupling, the synchronization would effect assimilation of Fourier-space data from the  $A$  channel into the  $B$  channel.

Since the problem of data assimilation arises in any situation requiring a computational model of a parallel physical process to track that process as accurately as possible based on limited input, it is suggested here that the broadest view of data assimilation is that of machine perception by an artificially intelligent system. Like a data assimilation system, the human mind forms a model of reality that functions well, despite limited sensory input, and one would like to impart such an ability to the computational model.

In this more general context, the role of synchronism is indeed reminiscent of Jung’s notion of synchronicity in the relationship between mind and the material world. Noting that standard data assimilation approaches, if successful, do achieve synchronization, it only remains to determine whether there are synchronization schemes that lead to faster convergence than the standard data assimilation algorithms. It has been shown analytically that optimal synchronization is equivalent to Kalman filtering when the dynamics change slowly in phase space, so that the same linear approximation is valid at each point in time for the real dynamical system and its model. When the dynamics change rapidly, as in the vicinity of a regime transition, one must consider the full nonlinear equations and there are better synchronization strategies than Kalman filtering or ensemble Kalman filtering. The deficiencies of the standard methods, which are well known in such situations, are usually remedied by ad hoc corrections, such as “covariance inflation”<sup>2</sup>. In the synchronization view, such corrections can be derived from first principles<sup>1765</sup>.

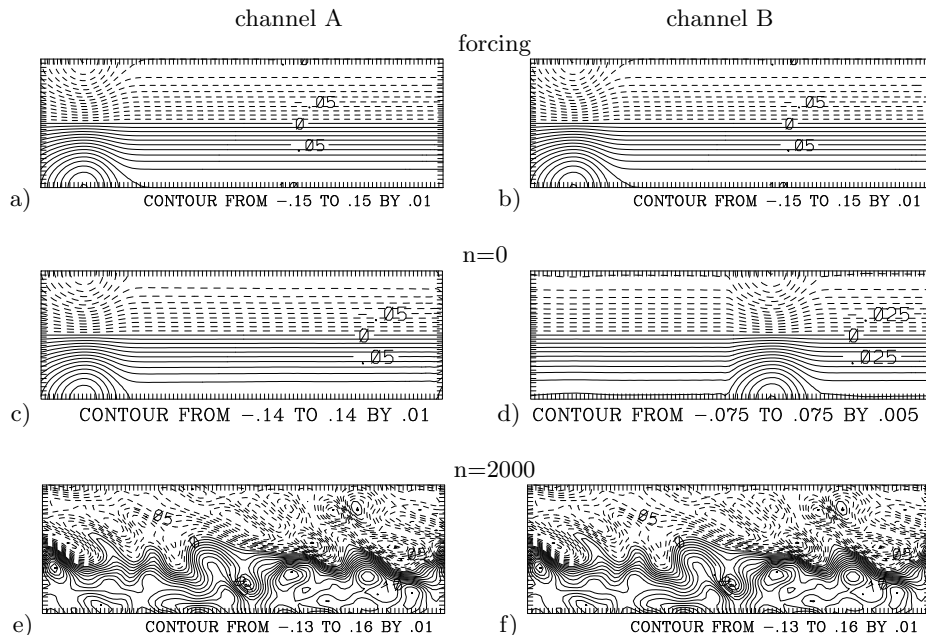


FIG. 4: Streamfunction (in units of  $1.48 \times 10^9 m^2 s^{-1}$ ) describing the forcing  $\psi^*$  (a,b), and the evolving flow  $\psi$  (c-f), in a parallel channel model with coupling of medium scale modes for which  $|k_x| > k_{x0} = 3$  or  $|k_y| > k_{y0} = 2$ , and  $|k| \leq 15$ , for the indicated numbers  $n$  of time steps in a numerical integration (generalizing to bidirectional coupling, for convenience). Parameters are as described previously<sup>15</sup>. An average streamfunction for the two vertical layers  $i = 1, 2$  is shown. Synchronization occurs by the last time shown (e,f), despite differing initial conditions.

#### IV. INTERNAL SYNC VS. MIND-MATTER SYNC: THE IMPORTANCE OF MEANING

In the Jungian version of synchronicity, as in the popular culture surrounding the topic, the alleged relationships between events, mental or physical, are detected without close inspection and are “meaningful”. To assess the promise of the dynamical systems paradigm as grounding for philosophical synchronicity, one needs to introduce a notion of meaning in the context of coupled dynamical systems, and to show that the requirement for such a property is typically satisfied. In this regard, the physical interpretation of the modelled phenomena is crucial.

Prior use of the idealized geophysical model of the last section suggests how “meaning” might enter. The quasi-geostrophic channel model was originally developed to represent the geophysical “index cycle”, in which the large-scale mid-latitude atmospheric circulation vacillates, at apparently random intervals, between two types of flow<sup>58</sup>. In the “blocked flow” regime, e.g. Fig. 5a, a large high-pressure center, typically over the Pacific or Atlantic, interrupts the normal flow of weather from west to east and causes a build-up of extreme conditions (droughts, floods, extreme temperatures) downstream. In the “zonal flow” regime, e.g. Fig. 5b, weather patterns progress normally. Synchronization of flow states, complete or partial, implies correlations between the regimes occupied by two coupled channel models at any given time. Such correlations, in a truth/model context, are indeed meaningful to meteorologists and to the residents of the regions downstream of any blocks. Synchronization of two highly simplified versions of the channel model has been used to predict correlations between blocking events in the Northern and Southern hemispheres<sup>11,12</sup>, and synchronization of two channel models has been used to infer conditions under which Atlantic and Pacific blocking events can be expected to anticorrelate<sup>14,15</sup>.

To generalize the above comments regarding meaningful synchronization in the geophysical models, we note that blocks are “coherent structures”, as commonly arise in a variety of nonlinear field theories. Such structures, of which solitons are perhaps the best known example, persist over a period of time because of a balance between nonlinear and dispersive effects. If two coupled systems exhibit synchronization of their detailed states, i.e. field configurations, then occurrences of corresponding coherent structures in the two systems will certainly correlate, and such coincidences will likely be interpreted as meaningful.

While no generally accepted definition of “coherent structure” has been articulated, one view of their fundamental nature can support the proposed general connection with meaningfulness. For a structure to persist, the different degrees of freedom of the underlying field theory must continue to satisfy a fixed relationship as they evolve separately. That behavior defines generalized synchronization, the phenomenon in which two dynamical systems synchronize, but with a correspondence between states given by a relationship other than the identity<sup>47</sup>. Coherent structures are then

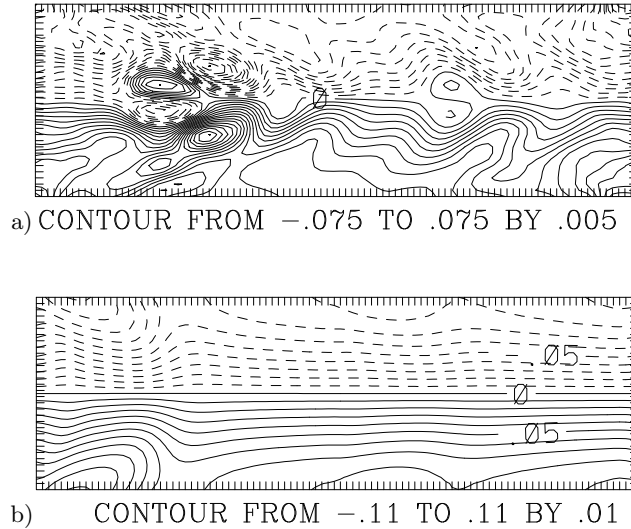


FIG. 5: Streamfunction (in units of  $1.48 \times 10^9 m^2 s^{-1}$ ) describing a typical blocked flow state (a) and a typical zonal flow state (b) in the two-layer quasigeostrophic channel model. An average streamfunction for the two vertical layers  $i = 1, 2$  is shown.

characterized by *internal* generalized synchronization within a system.

Consider two synchronously coupled systems, representing reality and model/mind, respectively, and suppose that each system also exhibits internal synchronization of some degrees of freedom. Then a knowledge of the current value of one of the internally synchronized variables in reality will automatically determine the values of all variables that synchronize with it. It is proposed that such a relationship captures “meaningfulness” in the usual sense of that term. The more extensive the internal synchronization pattern, the greater is the meaning of any single variable belonging to the pattern, and the greater is the meaning of any coincidence between the value of that variable and the corresponding value in another system. But such coincidences are inevitable if the underlying fields are totally or intermittently synchronized.

Meaningfulness is even more naturally defined as internal synchronization within mind. A response to a given external stimulus by any “element” of mind is likely to be deemed meaningful if there are synchronized, parallel responses of other mental elements.

It remains to show that internal synchronization is required or likely in each of a pair of dynamical systems that exhibit synchronized chaos. Such a requirement has recently been hypothesized<sup>20</sup>. The existence of internal synchronization clearly facilitates synchronization with an external system - coupling any pair of corresponding variables each of which belong to an internally synchronized group will tend to synchronize the other members of each group with their counterparts in the other system. The new hypothesis is that the existence of such groups is implied by the possibility of external synchronization, and thus that meaningful coherent structures are likely wherever loosely coupled chaotic systems are found to synchronize.

The hypothesis was corroborated in previous work on a type of coherent structure that may be essential in cosmological models - “oscillons” in a nonlinear scalar field theory. These are structures in the field that oscillate in fixed, randomly placed locations, as do similar structures that were first noted in vibrating piles of sand<sup>56</sup>. The expansion of the universe plays a role in the cosmological case that is analogous to the role of frictional dissipation in the sandpiles. An idealized one-dimensional model is given by the relativistic scalar field equation, with a nonlinear potential term, in an expanding background geometry described by a Robertson-Walker metric with Hubble constant  $H$ . Using covariant derivatives for that metric in place of ordinary derivatives, one obtains the field equation

$$\partial^2 \phi / \partial t^2 + H \partial \phi / \partial t - e^{-2Ht} \partial^2 \phi / \partial x^2 + V'(\phi) = 0 \quad (4)$$

The scalar field exhibits oscillon behavior for some forms of the nonlinear potential  $V$  (Fig. 6a), but not for others (Fig. 6b).

Where oscillons exist, a crude form of synchronized chaos is observed for a pair of loosely coupled scalar field systems (a configuration that is introduced to study the synchronization patterns, without physical motivation), as seen in Fig. 7. The fields do not synchronize, but the oscillons in the two systems tend to form in the same locations. For a potential that does not support oscillons, such positional coincidence is trivially absent, and there is no correlation

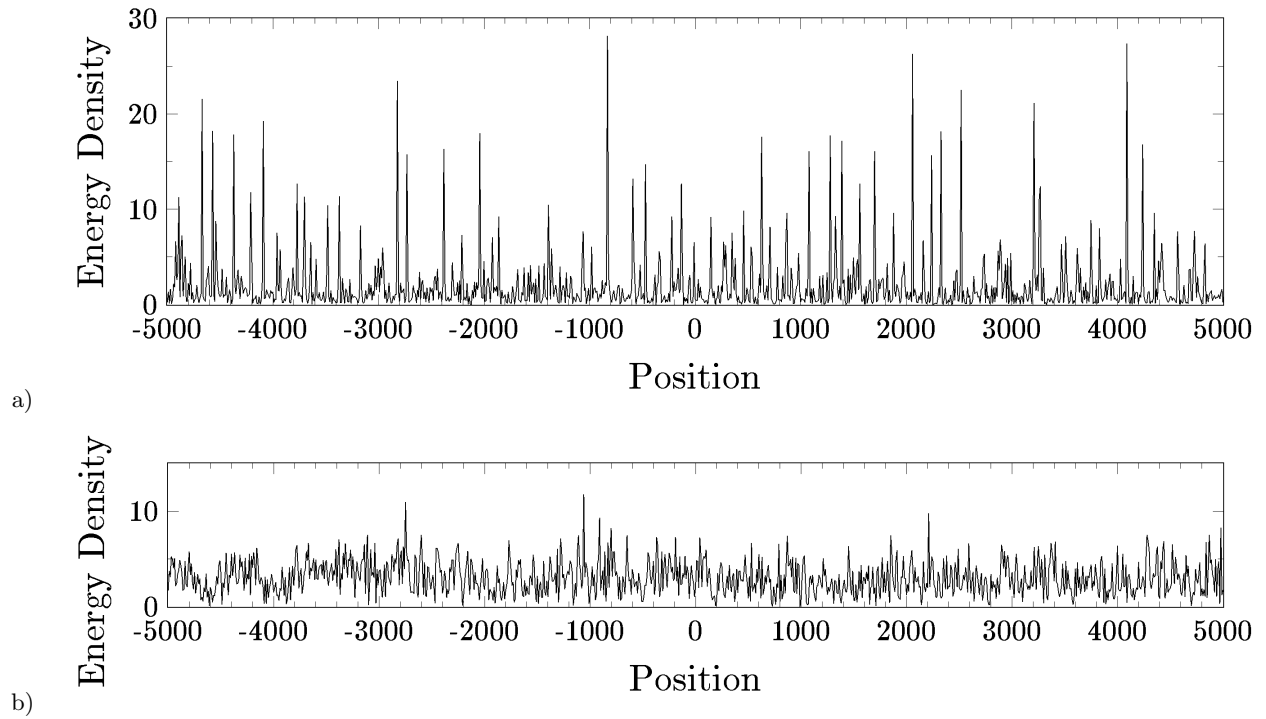


FIG. 6: Energy density  $\rho = (1/2)e^{-Ht}(\phi_x)^2 + (1/2)e^{Ht}(\phi_t)^2 + e^{Ht}V(\phi)$  vs. position  $x$  for a numerical simulation of the scalar field equation (4), suggesting localized oscillons (a), and a simulation of the same equation, but with a different potential  $V(\phi) = (1/2)\phi^2 + (1/4)\phi^4 + (1/6)\phi^6$ , for which oscillons do not occur (b).

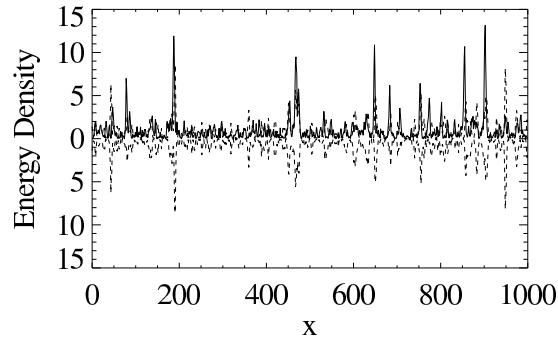


FIG. 7: The local energy density  $\rho$  vs.  $x$  for two simulations of the scalar field equation (4), coupled to one another only through modes of wavenumber  $k \leq 64$  at final time. ( $\rho$  for the second system (dashed line) is also shown reflected across the  $x$ -axis for ease in comparison.) The coincidence of oscillon positions is apparent.

between corresponding components of the underlying field. Synchronization in this case can *only* be interpreted in terms of coherent structures in the separate systems.

In a system as simple as the 3-variable Lorenz model, the hypothesis about the relationship between internal and external synchronization is also validated. In this case the relationship gives insight about which variables can be coupled to give synchronized chaos. Along the Lorenz attractor, the variables  $X$  and  $Y$  partially synchronize, resulting in the near-planar shape, while  $Z$  is independent. Consistently with the hypothesized relationship, either  $X$  or  $Y$ , but not  $Z$ , can be coupled to the corresponding variable in an external system to cause the two systems to synchronize.

To summarize the conclusions of this section: The meaningfulness of a synchronization pattern is naturally defined in terms of internal synchronization, or coherent structures, involving some of the variables that synchronize externally. But external synchronization usually implies the existence of internal synchronization, and hence meaning.



## V. SYNC AS AN ORGANIZATIONAL PRINCIPLE IN COMPUTATIONAL MODELING

If chaos synchronization provides a rational foundation for philosophical synchronicity, it should give deeper insight regarding apparent synchronicity in physical and psychological phenomena and underlying mechanisms. In the psychological realm, it has already begun to appear that synchronized oscillations play a key role. Synchronized firing of neurons has been introduced as a mechanism for grouping of different features belonging to the same physical object<sup>24,48,59</sup>. Recent debates over the physiological basis of consciousness have centered on the question of what groups or categories of neurons must fire in synchrony in a mental process for that process to be a “conscious” one<sup>33</sup>. Here we argue, based on the behavior of specific dynamical systems, that a) patterns of synchronized firing of neurons in the brain, that are chaotically intermittent, provide a particularly natural and useful representation of objective grouping relationships; and b) the role of synchronization in consciousness may be a natural extension of its role in perception as described in Section III.

### A. Representational Utility of Sync

A first stage in perception is the organization of raw input data into distinct groups, to which meaning can later be assigned. In the case of visual processing by brains or by computers, the grouping problem is typically that of the “segmentation” of the field of view into a small number of connected components, corresponding to physically distinct entities. It has been suggested that in biological systems, the “binding” of features sensed by different neurons is accomplished by the brief synchronization of the firing (spike trains) of those neurons<sup>60</sup>. In machine vision, segmentation defines a finite combinatorial optimization problem for an image consisting of discrete pixels. One might therefore imagine an artificial system consisting of an array of oscillators that form patterns of internally synchronized groups, each group corresponding to a different image segment<sup>53</sup>.

The utility of such a representation can be explored in a “cellular neural network”, which generalizes old-style artificial neural networks so that each unit in the network is a regular oscillator instead of a fixed input-output function<sup>10</sup>. A synchronized-oscillator representation was indeed used in previous work<sup>21</sup> to solve a more difficult combinatorial optimization problem - the traveling salesman problem, cast in a form originally described by Hopfield and Tank<sup>27</sup>. The conclusion was that synchronicity-as-synchronized-chaos likely plays a key role in allowing networks of chaotic oscillators to reach globally optimal solutions.

In the original Hopfield-Tank scheme, a tour among cities is represented as a matrix of binary values, as in Fig. 8a, which represents the tour ECABD. With one “neural” unit at each location in the matrix, network dynamics are introduced that drive a cost-function to local minima. The cost-function, in terms of the real-valued  $V_{Xi}$  (which are dynamically driven to binary values), is

$$E = A/2 \sum_X \sum_i \sum_{j \neq i} V_{Xi} V_{Xj} + B/2 \sum_i \sum_X \sum_{Y \neq X} V_{Xi} V_{Yi} + C/2 \left[ \left( \sum_X \sum_i V_{Xi} \right) - n \right] \\ + D/2 \sum_X \sum_{Y \neq X} \sum_i d_{XY} V_{Xi} (V_{Y,i+1} + V_{Y,i-1}) \quad (5)$$

where the summation indices  $X$  and  $Y$  range over the  $n$  cities (rows), and the indices  $i$  and  $j$  range over the  $n$  slots in the schedule (columns). The last term includes the pairwise inter-city distances  $d_{XY}$  and the other terms tend to ensure a sparse pattern of the proper form, with only one unit “on” in each row (“A” term), only one in each column (“B” term), and  $n$  in total (“C” term).

In the CNN scheme, the same tour as above is represented in each of 5 groups of synchronized oscillators, as in Fig. 8b, where the redundant representations are cyclic permutations of the tour cities. The cost-function that is minimized is now given in terms of complex variables  $z_{Xi}$  with phases corresponding to the relative phases of the oscillators. The cost function is

$$L = A \sum_{ij} (|z_{ij}|^2 - 1)^2 + B \sum_{ij} |(z_{ij}/|z_{ij}|)^n - 1|^2 - C \sum_i \sum_{jj'} |z_{ij}/|z_{ij}| - z_{ij'}/|z_{ij'}||^2 \\ - D \sum_j \sum_{ii'} |z_{ij}/|z_{ij}| - z_{i'j}/|z_{i'j}||^2 + E \sum_i \sum_j \sum_{j'} d_{jj'} \text{Re}(z_{ij} z_{i+1,j'}^* / |z_{ij}| |z_{i+1,j'}|). \quad (6)$$

If the oscillators are governed by  $\dot{z}_{ij} = -\frac{\partial L}{\partial z_{ij}^*}$ ,  $\dot{z}_{ij}^* = -\frac{\partial L}{\partial z_{ij}}$ , and a simulated annealing regimen is introduced, the CNN will converge to solutions as shown in Fig. 9.

The performance of the oscillator network is surprising because the selection task is much more difficult: There are 5! possible assignments of phase states to the 5 copies of a tour that are represented on the matrix, and 2 possible tour

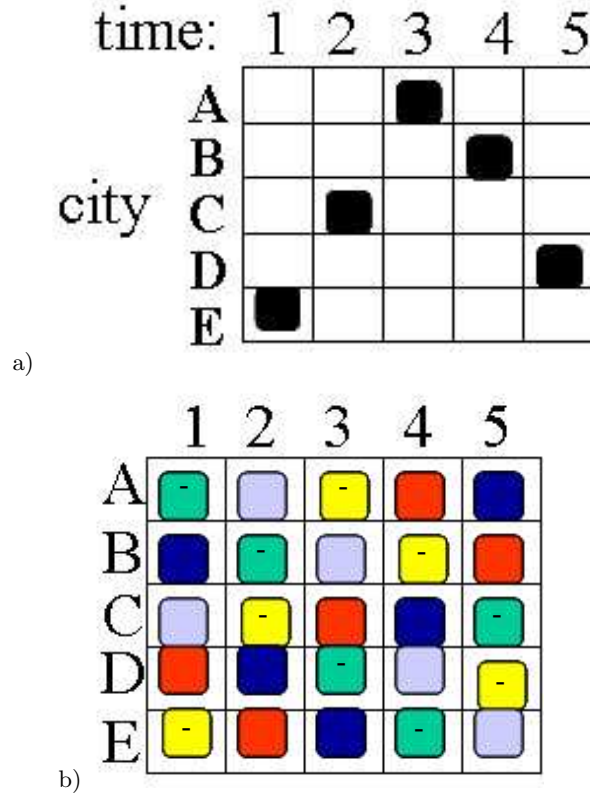


FIG. 8: In the original Hopfield representation (a), a tour (here ECABD) is specified by the collection of units that are “on”, provided there is only one “on” unit in each row and only one in each column. In the CNN representation (b) equivalent cyclic permutations of the same tour are simultaneously represented as sets of 5 units that are synchronized, with an analogous proviso. Units with the same relative phase are shown in the same color. (Dash marks distinguish ambiguous colors in a grayscale rendition.)

directions, so the architecture is effective in selecting  $2 \times 5! = 240$  shortest-distance tours out of  $5^{25}$  possible states. For the original Hopfield architecture, the only arbitrariness is in the choice of starting city and tour direction, so that architecture would select  $2 \times 5 = 10$  binary-valued tour states out of  $2^{25}$  possibilities. The task of the synchronization network is more difficult by a factor of  $\frac{10}{240} \left(\frac{5}{2}\right)^{25} = 3.7 \times 10^8$ , a factor which grows exponentially with the number of cities.

If the regular oscillators are replaced by chaotic oscillators, then intermittent desynchronization can play the same role as simulated annealing and may do so in a perceptual grouping context as well. Solutions would occur as brief internal “synchronicities” in the network. It should be recalled that in biological systems, periods of synchronization remain very brief, typically lasting less than 200 milliseconds.

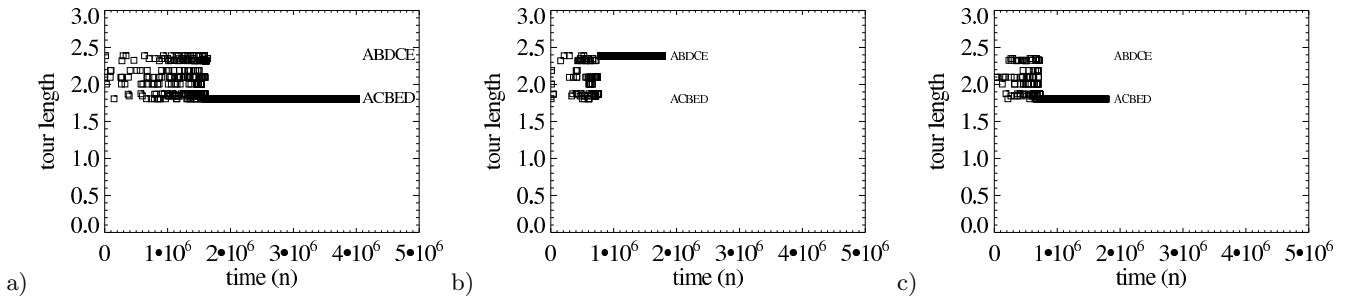


FIG. 9: Network histories for three runs of the cellular neural network with architecture and annealing schedules as described previously<sup>21</sup>. Runs shown are for slow annealing (a) and fast annealing (b,c), with randomly chosen initial conditions. Only states (squares) corresponding to tours are plotted, with tour lengths as indicated.

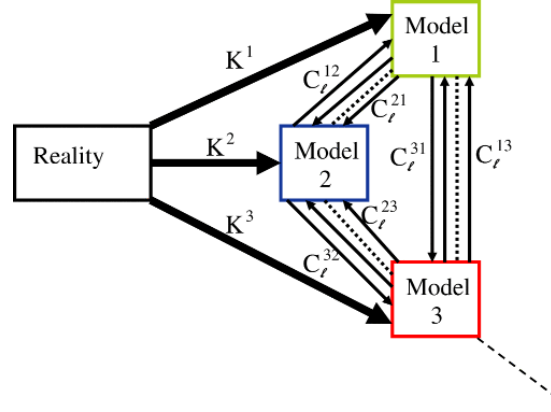


FIG. 10: “Model” Lorenz systems are linked to each other, generally in both directions, and to “reality” in one direction. Separate links between models, with distinct values of the connection coefficients  $C_l^{ij}$ , are introduced for different variables and for each direction of possible influence.

The advantage of the synchronization-based representation is that it affords a particularly loose form of feature binding, that allows a network to search over alternative groupings until a globally optimal one is found. Freeman<sup>23</sup> has suggested that the general role for chaos in neural information processing is to provide a combination of sensitivity and stability that might account for the observed range of mental competence in the face of unique inputs. Internal synchronicity in neural systems, realized as highly intermittent synchronization of chaos, provides an explicit mechanism for the suggested role.

### B. Model Fusion

The role of synchronization of the neuronal 40Hz oscillation in perceptual grouping has led to speculations about the role of synchronization in consciousness<sup>33,46,51,59</sup>, but here we suggest a relationship on a more naive basis: Consciousness can be framed as self-perception, and then placed on a similar footing as perception of the objective world. In this view, there must be semi-autonomous parts of a “conscious” mind that perceive one another. In the interpretation of Section III, these components of the mind synchronize with one another, or in alternative language, they perform “data assimilation” from one another, with a limited exchange of information.

Taking this interpretation of consciousness seriously, again imagine that the world is a 3-variable Lorenz system, perceived by three different components of mind, also represented by Lorenz systems, but with different parameters. The three Lorenz systems also “self-perceive” each other. Three imperfect “model” Lorenz systems were generated by perturbing parameters in the differential equations for a given “real” Lorenz system and adding extra terms. The resulting suite is:  $\dot{x} = \sigma(y - z)$ ,  $\dot{y} = \rho x - y - xz$ ,  $\dot{z} = -\beta z + xy$

$$\begin{aligned}
 \dot{x}_i &= \sigma_i(y_i - z_i) + \sum_{j \neq i} C_{ij}^x(x_j - x_i) + K_x(x - x_i) \\
 \dot{y}_i &= \rho x_i - y_i - x_i z_i + \mu_i + \sum_{j \neq i} C_{ij}^y(y_j - y_i) + K_y(y - y_i) \\
 \dot{z}_i &= -\beta_i z_i + x_i y_i + \sum_{j \neq i} C_{ij}^z(z_j - z_i) + K_z(z - z_i)
 \end{aligned} \tag{7}$$

where  $(x, y, z)$  is the real Lorenz system and  $(x_i, y_i, z_i)$   $i = 1, 2, 3$  are the three models. An extra term  $\mu$  is present in the models but not in the real system. Because of the relatively small number of variables available in this toy system, all possible directional couplings among corresponding variables in the three Lorenz systems were considered, giving 18 connection coefficients  $C_{ij}^A$   $A = x, y, z$   $i, j = 1, 2, 3$   $i \neq j$ . The constants  $K_A$   $A = x, y, z$  are chosen arbitrarily so as to effect “data assimilation” from the “real” Lorenz system into the three coupled “model” systems. The configuration is schematized in Fig. 10.

The connections linking the three model systems were chosen using a machine learning scheme that follows from a general result on parameter adaptation in synchronously coupled systems with mismatched parameters: If two systems synchronize when their parameters match, then under some weak assumptions it is possible to prescribe a dynamical evolution law for general parameters in one of the systems so that the parameters of the two systems, as well as the

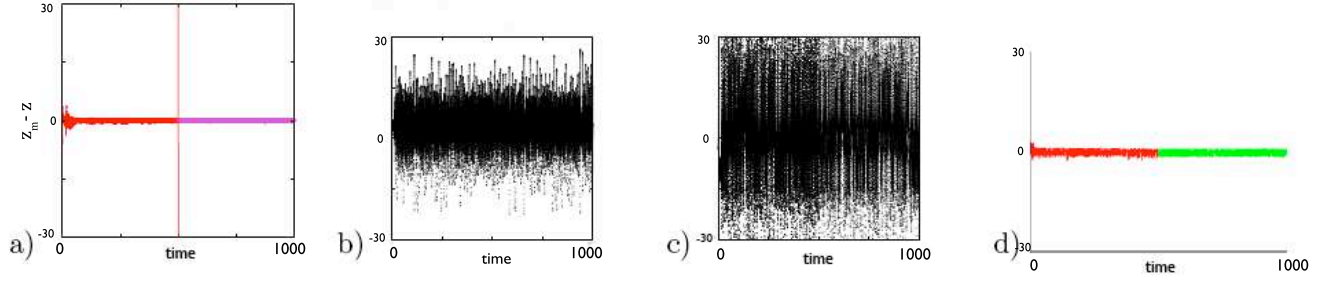


FIG. 11: Difference  $z_m - z$  between “model” and “real”  $z$  vs. time for a Lorenz system with  $\rho = 28$ ,  $\beta = 8/3$ ,  $\sigma = 10.0$  and a suite of models with  $\rho_{1,2,3} = \rho$ ,  $\beta_1 = \beta$ ,  $\sigma_1 = 15.0$ ,  $\mu_1 = 30.0$ ,  $\beta_2 = 1.0$ ,  $\sigma_2 = \sigma$ ,  $\mu_2 = -30.0$ ,  $\beta_3 = 4.0$ ,  $\sigma_3 = 5.0$ ,  $\mu_3 = 0$ . The synchronization error is shown for a) the average of the coupled suite  $z_m = (z_1 + z_2 + z_3)/3$  with couplings  $C_{ij}^A$  adapted according to (8) for  $0 < t < 500$  and held constant for  $500 < t < 1000$ ; b) the same average  $z_m$  but with all  $C_{ij}^A = 0$ ; c)  $z_m = z_1$ , the output of the model with the best  $z$  equation, with  $C_{ij}^A = 0$ ; d) as in a) but with  $\beta_1 = 7/3$ ,  $\sigma_2 = 13.0$ , and  $\mu_3 = 8.0$ , so that no equation in any model is “correct”. (Analogous comparisons for  $x$  and  $y$  give similar conclusions.)

states, will converge<sup>19</sup>. In the present case the tunable parameters are taken to be the connection coefficients (not the parameters of the separate Lorenz systems), and they are tuned under the peculiar assumption that reality itself is a similar suite of connected Lorenz systems. The general result<sup>19</sup> gives the following adaptation rule for the couplings:

$$\dot{C}_{i,j}^x = a(x_j - x_i) \left( x - \frac{1}{3} \sum_k x_k \right) - \epsilon / (C_{i,j}^x - 100)^2 + \epsilon / (C_{i,j}^x + \delta)^2 \quad (8)$$

with analogous equations for  $\dot{C}_{i,j}^y$  and  $\dot{C}_{i,j}^z$ , where the adaptation rate  $a$  is an arbitrary constant and the terms with coefficient  $\epsilon$  dynamically constrain all couplings  $C_{i,j}^A$  to remain in the range  $(-\delta, 100)$  for some small number  $\delta$ . Without recourse to the general result on parameter adaptation, the rule (8) has a simple interpretation: Time integrals of the first terms on the right hand side of each equation give correlations between truth-model synchronization error, e.g.  $x - \frac{1}{3} \sum_k x_k$ , and inter-model “nudging”, e.g.  $x_j - x_i$ . We indeed want to increase or decrease the inter-model nudging, for a given pair of corresponding variables, depending on the sign and magnitude of this correlation. (The learning algorithm we have described resembles a supervised version of Hebbian learning. In that scheme “cells that fire together wire together.” Here, corresponding model components “wire together” in a preferred direction, until they “fire” in concert with reality.) The procedure will produce a set of values for the connection coefficients that is at least locally optimal.

A simple case is one in which each of the three model systems contains the “correct” equation for only one of the three variables, and “incorrect” equations for the other two. The “real” system could then be formed using large connections for the three correct equations, with other connections vanishing.

In a numerical experiment (Fig. 11a), the couplings did not converge, but the coupled suite of “models” did indeed synchronize with the “real” system, even with the adaptation process turned off half-way through the simulation so that the coupling coefficients  $C_{i,j}^A$  subsequently held fixed values. The difference between corresponding variables in the “real” and coupled “model” systems was significantly less than the difference using the average outputs of the same suite of models, not coupled among themselves. (With the coupling turned on, the three models also synchronized among themselves nearly identically, so the average was nearly the same in that case as the output of any single model.) Further, without the model-model coupling, the output of the single model with the best equation for the given variable (in this case  $z$ , modeled best by system 1) differed even more from “reality” than the average output of the three models. Therefore, it is unlikely that any *ex post facto* weighting scheme applied to the three outputs would give results equalling those of the synchronized suite. Internal synchronization within the multi-model “mind” is essential. In a case where no model had the “correct” equation for any variable, results were only slightly worse (Fig. 11d).

The connections found by the adaptation procedure (8) are only locally optimal. For the configuration with the results depicted in Fig. 11a-c, the adapted coefficients should be binary-valued (i.e. 0 or 100), in order to select the “correct” equation for each variable and ignore the other equations. Instead, the coefficients are as listed in Table I, with almost no trace of the expected pattern.

It is thought that the addition of a stochastic component or chaos in the training stage would allow the connection coefficients to reach a global optimum, e.g. the pattern of binary values indicated by the asterisks in the Table. Such a result would correspond to a complete fusion of the models, rather than a weak (though satisfactory) “consensus”.

TABLE I: Connection coefficients as determined by the adaptive coupling scheme. In the globally optimal configuration, values marked with \* should be large; others should vanish.

	$C_{12}$	$C_{13}$	$C_{21}$	$C_{23}$	$C_{31}$	$C_{32}$
$C_{ij}^x$	2.7*	2.7	2.3	2.4	2.5	3.5*
$C_{ij}^y$	2.6	2.5*	2.4	2.2*	2.9	2.7
$C_{ij}^z$	1.9	11.1	5.9*	3.2	4.4*	2.3

In the realm of computational modeling, the inter-model synchronizaiton scheme provides a way to combine a suite of imperfect models of the same objective process, requiring only that the models come equipped with a procedure to assimilate new measurements from the process in real time, and hence from one another. The scheme has indeed been suggested for the combination of long-range climate projection models, which differ significantly among themselves in regard to the magnitude and regional characteristics of expected global warming<sup>18</sup>. In this context, results have been confirmed and extended with the use of a more developed machine learning scheme to determine inter-model connections<sup>57</sup>. The scheme could also be applied to financial, physiological, or ecological models.

The facility with which alternative models synchronize is taken here to bolster the previous suggestions that synchronization plays a fundamental role in conscious mental processing. It remains to integrate a theory of higher-level synchronization with the known synchronization of 40Hz spike trains. It is certainly plausible that inter-scale interactions might allow synchronization at one level to rest on and/or support synchronization at the other level. Inter-scale interactions played a similar role in the synchronization of a range of Fourier components of the same field in the synchronously coupled systems of partial differential equations considered in Section III. In a complex biological nervous system, with a steady stream of new input data, it is also very plausible that natural noise or chaos would give rise to very brief periods of widespread high-quality synchronization across the system, and possibly between the system and reality. Such “synchronicities” would appear subjectively as consciousness.

## VI. SYNC IN QUANTUM THEORY

In the realm of basic physics, the fundamental role of synchronism is most evident in the surprising long-distance correlations that characterize quantum phenomena. The Einstein-Podolsky-Rosen (EPR) phenomenon, viewed in light of Bell’s Theorem, implies that spatially separated physical systems with a common history continue to evolve as though connected with each other and with observers. As with synchronized chaos, quantum entanglement can be used for cryptography, an analogy that was developed in prior work<sup>13</sup>. Bell’s Theorem definitively asserts that observed correlations cannot be attributed only to shared initial conditions. One may naturally seek an interpretation of EPR correlations in terms of synchronized chaotic oscillators, if one puts quantum theory on a non-local deterministic footing, as in Bohm’s causal interpretation<sup>6,7</sup>, and as suggested more recently by ’t Hooft<sup>55</sup>. The condition that these connections not give rise to supraluminal transmission of information restricts the form of such a theory. In this Section, which is somewhat more speculative than the preceeding ones, we extend a previous argument<sup>16</sup> that quantum nonlocality can possibly arise from chaos synchronization, but probably requires a multiply connected space-time geometry.

The EPR phenomenon occurs with a pair of spin-1/2 particles that are produced by the decay of a spinless particles and then separate by a large distance. By conservation of angular momentum, the two particles must have opposite spins. Thus a measurement of one particle’s spin, along any axis, conveys knowledge of the spin of the second particle as well, that can be confirmed by observation. In the standard (Copenhagen) interpretation of quantum theory, the measured spin values do not exist prior to measurement, and so the measurement must affect the spin of the opposite particle, even if the separation between the particles is space-like and such affect would require faster-than-light signalling. An interpretation in which the spin values, for any possible choice of measurement axis, belong to the particles prior to measurement naively seems preferable. But Bell’s Theorem<sup>5</sup> precludes such an interpretation, by falsifying a property of the spin correlations that would follow from it. Bell’s inequality for two spin-1/2 particles in an entangled quantum (singlet) state follows from the assumption that the measured binary-valued spins  $A$  and  $B$  are only functions of the orientations  $a$  and  $b$  of the respective measuring devices, and of some hidden variables represented collectively by  $\lambda$ , i.e.  $A = A(a, \lambda)$ ,  $B = B(b, \lambda)$ . In general,  $\lambda$  designates the state of the joint system at some initial time. One then defines the correlation  $P$  between the two measured spins as a function of the two orientations  $a$  and  $b$ ,  $P(a, b) \equiv \int \rho(\lambda) A(a, \lambda) B(b, \lambda) d\lambda$ , where  $\rho(\lambda)$  is any function specifying a probability distribution

of the hidden variables, i.e.  $\int \rho(\lambda) d\lambda = 1$ . One introduces a third orientation  $b'$ , and considers the difference  $P(a, b) - P(a, b') = -\int d\lambda \rho(\lambda) [A(a, \lambda)A(b, \lambda) - A(a, \lambda)A(b', \lambda)] = \int d\lambda \rho(\lambda) A(a, \lambda)A(b, \lambda)[A(b, \lambda)A(b', \lambda) - 1]$  having used the convention  $A = \pm 1$ ,  $B = \pm 1$ , and having substituted  $B(b, \lambda) = -A(b, \lambda)$ . It follows that  $|P(a, b) - P(a, b')| \leq \int d\lambda \rho(\lambda) [1 - A(b, \lambda)A(b', \lambda)]$ , which is Bell's inequality:

$$|P(a, b) - P(a, b')| \leq 1 + P(b, b') \quad (9)$$

The standard quantum theoretic result  $P(a, b) = -\cos(a - b)$  violates (9) (as does any smooth function), so the general hidden-variable form  $A = A(a, \lambda)$ ,  $B = B(b, \lambda)$ , based on spin systems that evolve independently after the initial decay, is ruled out.

The picture of the quantum world that emerges is one in which everything is entangled with everything else, in a web of relationships similar to the one implied by the ubiquity of chaos synchronization. Indeed, Palmer<sup>40</sup> has suggested that the quantum world lives on a dynamically invariant fractal point set within the higher-dimensional phase space associated with the degrees of freedom that are naively thought to be independent. Membership in the invariant set is an uncomputable property, so theories can only be formulated in terms of the variables of the full phase space. Palmer's invariant set is in fact a generalized synchronization "manifold" (the common but improper term, since the manifold is nowhere smooth), of the sort suggested by Fig. 2c. As discussed in prior work<sup>16</sup>, generalized rather than identical synchronization is the fundamental relationship because EPR spins *anti*-correlate.

To bar supraluminal transmission of information, we rely on the mechanism proposed for synchronization-based cryptography: a signal provided by one variable of a chaotic system is difficult to distinguish from noise and is meaningful only when received by an identical copy of the system. However, it follows from Takens theorem that information can be extracted from such a signal if one considers a long enough time series. Longer time series are required to decode signals produced by more complex systems. For perfect security, one would need a chaotic system with an infinite-dimensional attractor.

Such a situation would arise most naturally in a multi-scale system (e.g. as proposed by Palmer<sup>39</sup>) requiring at least a system of partial differential equations, but something can be learned by considering a family of simpler systems of variable dimension, given by ordinary differential equations<sup>13,16</sup>. It is known that two  $N$ -dimensional Generalized Rossler systems (GRS's) (each equivalent to a Rossler system for  $N = 3$ ) will synchronize for any  $N$ , no matter how large, when coupled via only one of the  $N$  variables:

$$\begin{aligned} \dot{x}_1^A &= -x_2^A + \alpha x_1^A + x_1^B - x_1^A & \dot{x}_1^B &= -x_2^B + \alpha x_1^B + x_1^A - x_1^B \\ \dot{x}_i^A &= x_{i-1}^A - x_{i+1}^A & \dot{x}_i^B &= x_{i-1}^B - x_{i+1}^B \quad i = 2 \dots N-1 \\ \dot{x}_N^A &= \epsilon + \beta x_N^A (x_{N-1}^A - d) & \dot{x}_N^B &= \epsilon + \beta x_N^B (x_{N-1}^B - d) \end{aligned} \quad (10)$$

Each system has an attractor of dimension  $\approx N - 1$ , for  $N$  greater than about 40, and a large number of positive Lyapunov exponents that increases with  $N$ . As  $N \rightarrow \infty$ , while the synchronization persists, the signal linking the two systems becomes impossible to distinguish from noise. It was shown previously<sup>13</sup> that an inequality analogous to (9) could be constructed by arbitrarily bisecting the phase space to define final states analogous to spin-up/spin-down, and using a GRS parameter as an analogue of measurement orientation. That inequality is in fact violated because of the connection between the systems, but a naive observer would expect it to hold because he is unable to distinguish the connecting signal from noise.

In Palmer's view, there is no connecting signal because the world never leaves the "invariant set" (although the dissipative character of gravitational interactions is assumed to play a role cosmologically in dynamically constraining the universe to motion on the invariant set in the first place)<sup>40</sup>. Here we inquire as to the nature of the required "restoring force" if small perturbations transverse to the synchronization manifold are conceived as physical.

The GRS is a questionable model of reality because its "metric entropy," the sum of the positive Lyapunov exponents,  $\sum_{h_i > 0} h_i$ , is constant as  $N \rightarrow \infty$ , and that its largest Lyapunov exponent  $h_{max} \rightarrow 0$  as  $N \rightarrow \infty$ . In other words, the higher the dimension, the less chaotic the system. Such behavior is suspect in a system intended to represent unpredictable quantum fluctuations. It is not known whether systems that are more chaotic than the GRS, but with attractors of arbitrarily high dimension, can be made to synchronize with loose coupling, but it seems likely that such synchronization behavior would be restricted. PDE systems where chaos synchronization has been demonstrated have dissipative behavior that effectively gives a low-dimensional attractor that is part of an "inertial manifold" or approximate inertial manifold<sup>15</sup>. Without such low-dimensional behavior, synchronization of such systems by coupling a small number of variables would be more difficult. A deterministic theory underlying quantum behavior, such as that suggested by Palmer<sup>39</sup>, would behave even more wildly than any PDE system.

Taking the GRS behavior as  $N \rightarrow \infty$  to be generic, one must reconcile its increasingly mild character with the requirement that the nonlocal "signal" be perfectly masked through chaos. It was suggested previously<sup>16</sup> that the

issue is resolved if the GRS is viewed as a spatially asymptotic description of an intrinsically faster dynamics in a highly curved space-time. For reference, recall that an object falling into a black hole is perceived by an observer at a distance from the hole as approaching the horizon with decreasing velocity, but never reaching it. If the physical system that the GRS describes lives in the vicinity of a micro- black hole or wormhole, the variables in the asymptotic description will be slowed, but the actual physical processes will be realistically violent, and can couple to each other through “signals” that are perfectly masked.

### A. Nonlocality from Wormholes

A Planck-scale foam-like structure in space-time was posited by Hawking<sup>25</sup> in the context of a procedure to quantize classical general relativity where that structure contributes significantly to a sum over alternative *Euclidean* space-time geometries (i.e. with signature ++++). Here we propose a role for microwormholes in ordinary Lorentzian (signature -+++ ) space-time. Such a suggestion is consistent with theoretical arguments<sup>8</sup> and experimental evidence<sup>1</sup> for fundamental granularity in space-time structure. As explained in the Appendix, microwormholes may arise in a variant of general relativity defined by equations that are generally covariant but scale-dependent, and a weak divergence that arises from the recirculation of virtual quanta through wormholes may be avoided if the wormholes are sufficiently narrow.

The systems that must synchronize are defined on two-dimensional horizons at the mouths of the wormholes. It is consistent with the holographic principle<sup>52,54</sup> that such 2D fields capture the essential information about the full three-dimensional systems. Synchronization of fields on 2D horizons is also consistent with suggestions that fields representing 2D turbulence in fluids, but not 3D turbulence, can be made to synchronize<sup>14</sup>.

If we stipulate, with Palmer, that the synchronization manifold is fundamental, because the physical world never leaves it, then no wormholes are needed: We have two dynamical systems defining an anticorrelated EPR pair,  $\dot{\mathbf{x}} = F(\mathbf{x})$ , and  $\dot{\mathbf{y}} = G(\mathbf{y})$ , with  $\mathbf{x} \in R^N$  and  $\mathbf{y} \in R^N$ . The dynamics are modified so as to couple the systems:

$$\dot{\mathbf{x}} = \hat{F}(\mathbf{x}, \mathbf{y}) \quad \dot{\mathbf{y}} = \hat{G}(\mathbf{y}, \mathbf{x}) \quad (11)$$

and there is some locally invertible function  $\Phi : R^N \rightarrow R^N$  such that  $\|\Phi(\mathbf{x}) - \mathbf{y}\| \rightarrow 0$  as  $t \rightarrow \infty$ . Then the coupled dynamics are also defined by the two autonomous systems

$$\dot{\mathbf{x}} = \hat{F}(\mathbf{x}, \Phi(\mathbf{x})) \quad \dot{\mathbf{y}} = \hat{G}(\mathbf{y}, \Phi^{-1}(\mathbf{y})) \quad (12)$$

without recourse to wormholes or any nonlocal connections, provided we know the badly behaved function  $\Phi$  exactly. Otherwise, we rely on the narrow width of the wormholes to prevent supraluminal transmission of matter or information. Diffraction effects preclude communication, except in highly symmetrical situations, as in EPR, where constructive interference might account for the needed nonlocal connections. The isolated character of such quantum “synchronicities” follows from the rarity of the required symmetrical context. Our wormholes are reminiscent of those in the original construction of Wheeler<sup>62</sup>, who suggested that lines of electric force are always closed if positive and negative charges are thus connected at the microlevel.

That connections through narrow ducts can be sufficient to synchronize spatially extended systems has already been demonstrated. Kocarev *et al.*<sup>32</sup> showed that pairs of PDE systems of various types (Kuramoto-Sivishinsky, complex Ginzburg-Landau, etc.) could be synchronized by pinning corresponding variables to one another at a discrete set of points, at discrete instants of time. (The example of synchronizing two quasigeostrophic channel models (Fig. 4) establishes essentially the same phenomenon for coupling formulated in Fourier space.) Wormholes of zero length are expected to give synchronization of subsystems on opposite sides in the same way. Intermittent synchronization could result if noise is introduced<sup>3</sup>, if the symmetrical situation is short-lived, or if the wormholes themselves are.

If the wormholes have finite length, then the resulting time lags will lead to a system of the same form as (2) that gives intermittent synchronization: The system (2) is indeed analogous to one derived from a pair of geophysical fluid models coupled by standing waves in narrow ducts<sup>11</sup>. Auxiliary variables analogous to  $S$  in (2) arise by first decomposing the field into a piece that satisfies the full nonlinear equations with homogeneous boundary conditions and a second piece that satisfies a linear system with matching boundary conditions in the region of the narrow ducts. The linear equations are solved using boundary Green’s functions that effect a time delay. The auxiliary variables are integrals of products of the boundary Green’s functions and differences of corresponding field variables from the two sides of the ducts.

The mediation of quantum interconnectedness by wormholes is perhaps the ultimate home for the oft-proposed marriage<sup>51</sup> between synchronization dynamics and small-world (or “scale-free”) networks. The question is essentially whether the construction can reproduce the nonlocal piece of the “quantum potential” in Bohm’s interpretation<sup>6</sup> (the remaining piece corresponding to motion along the synchronization manifold). The wormhole construction merits investigation, whether or not it turns out to be equivalent to less radical formulations.

## VII. SUMMARY AND CONCLUDING REMARKS

In the foregoing sections, we have attempted to show that the synchronization of loosely coupled chaotic systems approaches the philosophical notion of highly intermittent, meaningful synchronicity more closely than commonly thought. Synchronized chaos is highly intermittent in a natural setting (Section II). As with philosophical synchronicity, it describes the relationship between the objective world and a perceiving mind (Section III). Central to our thesis is a relationship between internal synchronization within a system, and external synchronizability with another physical system or with a model. That relationship, which was described in Section IV, is in accord with common wisdom: An objective system with a high degree of internal synchronization is more easily perceived/understood, an internally coherent individual can more easily engage the world, a nation that functions cohesively can play a greater role internationally, coherence in human society is accompanied by a more harmonious relationship with the physical environment, etc. In all such cases, the internal relationships, which are “meaningful” by our definition, would commonly be taken as meaningful in reality. Arguably, they are even more meaningful because of any synchronous relationship with the external world.

What is not clear is that even with the isolated character and meaningfulness of synchronicities in coupled chaotic systems, the phenomenon reaches all the way to that of Jung and others, who discussed detailed coincidences between physical events and previous dreams, for instance. The attempt to put relationships of that kind on a rational footing may appear doomed. The mechanisms of deterministic chaos seem insufficient. One may dismiss such examples and consider only more restricted forms of synchronicity, or one may imagine that new physical principles are at work. Philosophically, our endeavor might be compared to Marx’s attempt to ground Hegel’s dialectic in material reality, a transformation whose legitimacy has sometimes been questioned, notably by Bohm<sup>42</sup>. Even if our objective realization of Jung’s idea is only partial, it is the point of view of this paper that it is appropriate for scientists to seriously consider a concept that has captured the popular imagination as widely as has synchronicity, and to afford a rational explanation if possible.

The question is perhaps sharpest in regard to consciousness and synchronization-based theories thereof. In Section V, it was argued that previous suggestions about the role of synchronization in the brain were supported by the possibility of highly intermittent synchronization among chaotic oscillators and by the possibility of synchronizing different complex models of the same objective process, giving rise to “self-perception”. But Penrose has given a well known argument that the reasoning abilities of conscious beings cannot arise from classical physics or algorithmic processes that describe such physics: For any algorithmic system of ascertaining truth, one can always articulate a true statement, of the sort constructed by Gödel, that such a being knows to be true, but whose truth cannot be established within the system<sup>44</sup>. Since synchronized chaos is still deterministic<sup>66</sup>, the abilities of conscious beings must come from fundamentally different processes, which Penrose suggested are quantum mechanical.

Section VI was included above because quantum processes seem to provide the deepest example of synchronicity - the quantum world appears to live on a generalized synchronization “manifold”. But if Penrose is correct, the converse statement can also be made: Synchronicity as manifest in human consciousness is also fundamentally quantum in origin. Correlations in neuronal firing or between neural subsystems can only give rise to consciousness, in this view, if quantum correlations are involved. Synchronicities between states of the mind and of the objective world must somehow follow. Perhaps such an enlarged notion could reach the popular concept, and the one of Jung and Pauli. In any case, it seems likely that the question of the proper interpretation of quantum phenomena on the one hand, and that of the origin of synchronism between mind and matter on the other, will be resolved jointly.

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### Appendix A: On the possibility of microwormholes

As discussed in Section VI of the text, if space-time were permeated with micro-scale Wheelerian wormholes that would be useless for time travel, a synchronistic order might nonetheless emerge: chaos synchronization might combine with small-world effects in the manner that has been suggested for more restricted applications. Here we address two common objections to wormholes.

#### A.1 Wormholes are not proscribed by the weak-energy condition in higher-derivative gravity

While one might imagine that two Schwarzschild black hole solutions to Einstein’s equations could be joined to form a wormhole, solutions of this type are not traversible<sup>35</sup>. The possibility of traversible wormhole solutions is limited by



the *weak energy condition*. That condition states that for any null geodesic, say one parameterized by  $\zeta$ , with tangent vectors  $k^a = dx^a/d\zeta$ , an averaged energy along the geodesic must be positive:

$$\int_0^\infty T_{\alpha\beta} k^\alpha k^\beta > 0 \quad (\text{A1})$$

where  $T_{\alpha\beta}$  is the stress-energy tensor. Traversable wormholes can exist only if A1) is violated for some null geodesics passing through the wormhole, implying the existence of “exotic matter” with negative energy density in the “rest frame” of a light beam described by the null geodesic. The negative energy density is required, in one sense, to hold the wormhole open.

Quantum fluctuations in the vacuum can violate the weak energy condition<sup>9,36</sup>. But the problem might be avoided at the classical level, as desired if quantum theory is not to be presumed, if a larger class of generally covariant theories are considered. Terms containing higher derivatives of the metric can indeed be added to Einstein’s equations, with effects that are negligible on all but the smallest scales<sup>50,61</sup>. The situation is analogous to that of the Navier-Stokes equation in fluid dynamics: While the terms involving the co-moving derivative follow simply from Newton’s first law, the dissipative terms are ad hoc and can take many forms. General relativity can likewise be extended to theories of the form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + g_{\mu\nu}\Lambda + \sum_{n>2} c_n L^{n-2} R_{\mu\nu}^{(n)} = 8\pi T_{\mu\nu} \quad (\text{A2})$$

where  $R_{\mu\nu}^{(n)}$  is a quantity involving a total of  $n$  derivatives of the metric,  $L$  is a fundamental length scale, the  $c_n$  are dimensionless constants, and we have included a cosmological constant  $\Lambda$  for full generality. If  $L = L_P$ , the Planck length, then the new terms in the extended theory (A2) are negligible on macroscopic scales. They only need be considered if curvature is significant at the Planck length scale. Any metric that solves the ordinary Einstein equations after the substitution  $T_{\mu\nu} \rightarrow T_{\mu\nu} - (1/8\pi) \sum_{n>2} c_n L^{n-2} R_{\mu\nu}^{(n)}$  solves (A2) for given  $T_{\mu\nu}$ . It is plausible that the modified stress-energy tensor  $T_{\mu\nu} - (1/8\pi) \sum_{n>2} c_n L^{n-2} R_{\mu\nu}^{(n)}$  can be made to violate the weak energy condition if the signs of the constants  $c_n$  are chosen appropriately, and thus that a traversible micro-wormhole solution is possible.

## A.2 Damping of vacuum recirculation effects for narrow wormholes

The paradoxes usually associated with closed time-like curves that pass through wormholes have a quantum counterpart: Repeated passage of a virtual particle through a wormhole may lead to a divergence in  $T_{\mu\nu}$ . For each passage of a virtual particle through the wormhole, the contribution to the two-point function  $\langle \Psi | \hat{\phi}(x) \hat{\phi}(x') + \hat{\phi}(x') \hat{\phi}(x) | \Psi \rangle$  from a trajectory that contains that passage is attenuated by a factor  $b/D$ , where  $b$  is the wormhole width, and  $D$  is the spatial length of a geodesic through the wormhole, as measured in the frame of an “observer” traveling along the geodesic from the vicinity of  $x$  and  $x'$  through the wormhole once and back to the same vicinity. Here,  $x$  and  $x'$  are nearby points in space-time,  $|\Psi\rangle$  is the quantum state, and  $\hat{\phi}$  is the field operator associated with the field  $\phi$ . The contribution to the two-point function is found to behave as  $(b/D)^k \times 1/\sigma$ , where  $\sigma$  is  $1/2$  the square of the proper distance between  $x$  and  $x'$  along the geodesic connecting them through the wormhole, and the power  $k$  depends on the number of times that the trajectory traverses the wormhole. (Contributions from trajectories that traverse the wormhole only once dominate.) One finds  $\sigma \sim D\Delta t$ , where  $\Delta t$  is the proper time between  $x$  and the nearest null geodesic that passes through the wormhole. As  $x' \rightarrow x$ , the contribution diverges if  $x$  can be joined to itself by a null geodesic that passes through the wormhole. The stress-energy tensor can be expressed in terms of the two-point function<sup>31</sup> and also diverges as  $\sigma \rightarrow 0$  or  $\Delta t \rightarrow 0$ . Specifically, one finds  $T_{\mu\nu} \sim (b/D)^k \times 1/D(\Delta t)^3$  in natural units, or in dimensional units,

$$T_{\mu\nu} \sim (b/D)^k \times L_P/D \times m_P/(\Delta t)^3. \quad (\text{A3})$$

While we do not presume the validity of standard quantum theory in the present context, it might be helpful if the quantum recirculation divergence could be eliminated. For large scales on which the gravitational field itself need not be quantized, the theory that we wish to create must reproduce all verifiable predictions of standard quantum theory, and so agreement in the as-yet-to-be-verified domain of ordinary quantum theory in classical wormhole geometry would inspire confidence.

Kim and Thorne<sup>31</sup> argued that the divergence probably disappears in the proper quantum theory of gravity, allowing wormholes to remain. Quantization of the gravitational field in that theory would be effective on scales of  $L_P$ , the Planck length, so we only need consider the magnitude of  $T_{\mu\nu}$  for  $\sigma \geq L_P$ . At these scales, referring to (A3),  $T_{\mu\nu} \leq L_P/D$  in natural units of  $m_P/L_P^3$ , giving energy densities that are far too weak to destroy the

wormhole. Hawking<sup>26</sup>, in support of his “chrononology protection conjecture”, provided a counter-argument asserting that quantum gravity effects would only enter on much smaller scales, corresponding to the Planck length in the rest-frame of an “observer” travelling on one of the geodesics through the wormhole. The values attained by  $T_{\mu\nu}$  on scales larger than Hawking’s reduced length scale would still cause collapse of the wormhole, the instant that recirculation becomes possible.

Here, we note that there is an additional mechanism that could cut off the recirculation divergence for wormholes of very narrow width. Virtual particles of arbitrarily high energy cannot traverse the wormhole. High-energy virtual particles would reverse the effect of the exotic matter or of the higher-derivative terms, so the existence of the wormhole would again be precluded by the weak energy condition. The contribution to the energy flux from the virtual particles is  $T^{0i} = \frac{4}{\pi b^2} \int d\omega n(\omega) \hbar \omega$ , where  $n(\omega)$  is the number density of quanta at frequency  $\omega$ . (As in the Weizsacker-Williams approximation<sup>29</sup>, the quanta are assumed not to elongate in the wormhole.) At detailed resolution in frequency-space,  $n(\omega) = \sum_i n_i \delta(\omega - \omega_i)$ , where  $\omega_i$  is a discrete set of frequencies and  $n_i$  is a set of positive integers. There is a problem from the weak energy condition if any  $\omega_i > \omega_{cutoff}$  (with  $n_i \geq 1$ ), for  $\omega_{cutoff}$  sufficiently large as to cancel the negative-energy contributions to  $T^{00}$ . In a path integral, one need only consider histories in which more energetic particles either collapse the wormhole or are reflected and do not traverse it. In contrast, for wormholes of larger width, histories must be included in the path integral for which the energies of recirculating virtual quanta outside the wormhole are anomalously large. The cutoff implies that the term  $1/\sigma$  in the two-point function is replaced by a term like  $\int_{\omega_k < \omega_{cutoff}} d^4k \exp[ik \cdot (x - x')]/k^2$  which does not diverge, agreeing in this case with the original position of Kim and Thorne that vacuum polarization divergences are not an issue.

We note that highly intermittent wormhole behavior may still be enough to mediate long-range synchronization, in accordance with Kocarev *et al.*<sup>32</sup> findings for other types of PDE’s.

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- <sup>64</sup> No reference is made in this paper to the use of archetypes in physical theory or other aspects of Jung's philosophy.
- <sup>65</sup> The previous calculation of the covariance inflation factors<sup>17</sup> needs to be corrected to properly account for the discrete character of the actual assimilation cycle as contrasted with the continuous assimilation in the theoretical model.
- <sup>66</sup> If one considers a chaotic system given by differential equations for which infinite precision in initial conditions is needed to

predict the outcome even qualitatively, as in Palmer's earliest proposal<sup>38</sup>, a typical basin of attraction for a given outcome is a "fat fractal": The more precisely the initial conditions are known, the smaller is the probability of error in "guessing" the outcome. That is very unlike quantum indeterminacy.